



NCD-003-016201 Seat No. \_\_\_\_\_

**M. Sc. (Sem. II) (CBCS) Examination**

April / May - 2017

**Mathematics : CMT - 2001**

*(Algebra - II) (Old Course)*

**Faculty Code : 003**

**Subject Code : 016201**

Time :  $2\frac{1}{2}$  Hours]

[Total Marks : 70

- Instructions :** (1) Each question carries **equal** marks.  
(2) All questions are **compulsory**.

**1** Answer following short questions : **7×2=14**

- (i) For a ring  $R$ , define  $R$ -module  $M$  and  $R$ -submodule of  $M$ .
- (ii) Let  $M$  be an  $R$ -module. In Standard notation prove that  $(-a)m = -(am) = a(-m)$ ,  $\forall a \in R$  and  $\forall m \in M$ .
- (iii) Prove that  $x^3 + 3x + 2 \in \mathbb{Z}_7[x]$  is an irreducible polynomial over  $\mathbb{Z}_7$ .
- (iv) Write down the Eisenstein Criterion. Using it deduce that  $f(x) = x^3 + 5x^2 + 5x + 10 \in \mathbb{Q}[x]$  is irreducible polynomial over  $\mathbb{Q}$ .
- (v) For the field extension  $\mathbb{R}|Q$ , write down two elements of  $\mathbb{R}$  which are algebraic over  $Q$  and also write down two elements of  $\mathbb{R}$  which are transcendental elements over  $Q$  (They are not algebraic over  $Q$ ).
- (vi) Write down minimal polynomial of  $2^{\frac{1}{4}} \cdot i$  over  $Q$ .
- (vii) Prove or disprove  $Q\left(2^{\frac{1}{4}} \cdot i\right)/Q$  is a finite, normal and separable extension.

- 2** Attempt any two : **2×7=14**
- (a) Define a field extension. Let  $E/F$ ,  $K/E$  both are finite field extensions. Prove that  $K/F$  is also a finite field extension with  $[K : F] = [K : E] [E : F]$ .
- (b) Let  $F$  be a finite field. Prove that  $F^* = F - \{0\}$  is a cyclic group under multiplication.
- (c) Prove that  $Q(\sqrt{2}, \sqrt{3}, \dots, \sqrt{p}, \dots) | Q$  is an algebraic extension, but it is not a finite field extension.
- (d) Let  $E/F$ ,  $K/E$  both are algebraic field extensions. Prove that  $K/F$  is also an algebraic field extension.
- 3** Attempt any one : **1×14=14**
- (a) Let  $F$  be a finite field and  $|F| = p^n$ , for some prime  $p$  and  $n \in \mathbb{N}$ . Prove that  $\text{Aut}(F)$  is a cyclic group of order  $n$ .
- (b) State and prove Primitive Element Theorem.
- (c) Let  $E/F$  be a finite field extension. Prove that  $E/F$  is a simple extension iff there are only finite number of subfields of  $E$  that containing  $F$ .
- 4** Attempt any two : **2×7=14**
- (a) Let  $(N_i)_{i \in \Lambda}$  be a family of  $R$ -submodules of an  $R$ -module  $M$ . Prove that  $\bigcap_{i \in \Lambda} N_i$  is also an  $R$ -submodule of  $M$ .
- (b) Let  $f : M \rightarrow N$  be an  $R$ -homomorphism of  $R$ -modules. Prove that  $\text{Ker } f$  and  $f(M)$  are  $R$ -submodules of  $M$  and  $N$  respectively.
- (c) Let  $R$  be a ring with unity and  $M$  be an  $R$ -module. Prove that  $M$  is cyclic  $R$ -module iff  $M \simeq R/I$  for some left ideal  $I$  of  $R$ .
- (d) Let  $f : M \rightarrow N$  be an onto  $R$ -homomorphism of  $R$ -modules. In standard notation deduce that  $M/\text{Ker } f \simeq N$ .

5 Fill in the blanks appropriately : 7×2=14

(i) Let  $L/K, K/E$  be both are finite extensions. Let  $[L : K] = p,$

$[K : E] = n.$  What is value of  $[L : E] = \dots?$   $\{p + n, p^n,$

infinite, none of these}

(ii) Let  $F$  be a finite field. What is number of elements of  $F$  from followings is a possible case i.e.

$|F| = \dots? \{49, 50, 51, 100\}.$

(iii) Which of followings is a prime field ?

$\{Q, \mathbb{R}, \bar{Q}, \mathbb{C}\}.$

(iv) Let  $\omega = (-1, +\sqrt{3}i)/2.$  Which of following is the minimal

polynomial of  $\omega$  over  $Q$ ..... ?

$\{x^2 + x + 1, x^3 - 2, x^4 - 2, x^2 - 3\}.$

(v) Which of followings is a normal extension ?

$\left\{Q\left(2^{1/4}\right)|Q, Q\left(2^{1/3}\right)|Q, \bar{Q}|Q, \mathbb{C}|R\right\}.$

(vi) Which of followings is an irreducible polynomial in

$\mathbb{Z}_2[x]$ ..... ?

$\{x^2 + x, x^2 + 1, x^2, x^2 + x + 1\}.$

(vii) Which of followings is not a cyclic extension ?

$\left\{\mathbb{C}|R, Q\left(\cos\frac{2\pi}{p} + i\sin\frac{2\pi}{p}\right)|Q, Q\left(2^{1/4}, i\right)|Q\right\}.$

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